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COMPARATIVE ANALYSIS OF THE WORK OF BALL AND ROD MILLS ON THE BASIS OF THE PROBABILISTIC MODEL OF GRINDING OF MATERIALS

Abstract: The purpose of work is to map by the probability theory of grinding of processes in a rod mill with adaptation to this process of frequency, concentration, steric and activation factors. A comparative analysis of the grinding process in ball and rod mills based on the probabilistic model was carried out, during which it was found out that it is preferable to use rod mills for grinding larger fractions. This is achieved due to the advantage in the steric factor. For rod grinding in the whole range of grain sizes it exceeds that for ball grinding, due to less screening of grains by a rod than by a ball. At the same time, the activation factor also has a similar superiority, although to a lesser extent, which proves itself especially well for large fractions. The combined effect of steric and activation factors leads to the formation of maxima in the area of millimeter fractions. This maximum is much higher for rod grinding than for ball grinding. Due to this, the process of grinding by rods is theoretically much more efficient than ball grinding of large fractions, which practically corresponds to lower power consumption. The calculations have shown that in the case of rod grinding, the distribution of fractions is more uniform, and this also agrees with practical data. In rod grinding, as well as in ball grinding, a logarithmically normal distribution of fractions is formed as the process proceeds, which is related to the unity of applicability of the integral grinding model to any variants of sequential destruction of material. Due to consideration of all operating factors the received probabilistic model of rod grinding can be considered the most complete and ready for practical use.

Key words: probabilistic model, development, grinding, rod mills, ball mills, steric factor, activation factor, comparative analysis

Introduction. The waterfall mode, which creates the necessary chaotization of the balls and grains collisions to display by analogy with molecular kinetics, is realized in the operation of some rod mills [1,2], as well as in laboratory studies to prepare materials for flotation.

The general form of the probability equation of the grinding speed remains unchanged [3, 4]

$$-\frac{dP_j}{d\tau} = ZP_{\mathrm{st}}P_{\mathrm{a}}P_{conc}, \ \mathrm{s}^{-1}, \quad (1)$$

where Z - a frequency factor, P_{st} – a steric factor, P_a – an activation factor, $P_{conc} = P_b P_g P_j$ a concentration factor (P_b is volume fraction of balls, P_g – volume fraction of ore grains, P_j – fractional content of the j-th fraction).

However, formulas for steric (spatio-orientational) and activation (destructive) factors, previously obtained when balls are used as grinding bodies, require adaptation to operation of rod mills.

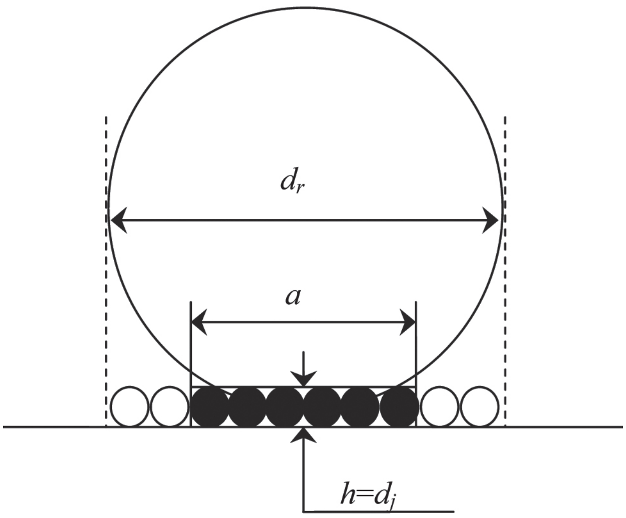
$$P_{\mathrm{st},b} = 4d_j\left(d_b - d_j\right)\Big/ d_b^2, \quad (2)$$

$$P_{a,b} = \exp\left(-\frac{E_a}{RT + \dfrac{3MmgD}{\pi d_j^3 \gamma_{\mathrm{g}}}}\right), \quad (3)$$

In these formulas d_b – a ball diameter, M – a molecular mass of the milled body, kg/mole; m – a mass of grinding body (ball), kg; E_a – activation energy of destruction of grains upon impact, J/mol; R – universal gas constant, J/(mol·K); T – the absolute temperature, K; g – acceleration of gravity, m/s^2; γ_g - density of the ground body (ore), kg/m^3.

Steric and activation factors counteract the effect of the size of balls and grains on them: the value P_{st} decreases as the size of the ball increases and the grain size decreases, which is the screening effect of this factor, which has the context of the probability that the grinding body collides with the ground one; on the contrary, the value P_a increases with the same change in the size of balls and grains, which determines the probability of grain destruction by the ball upon collision. The product of these factors takes into account the opposite nature of their joint influence on the grinding process, stipulating practically all the specifications of this process: its speed, energy efficiency and product quality by fractional composition [5]. It should be noted that the existing theoretical aspects of rod grinding have been developed only fragmentarily, without full consideration of all operating factors [6-10].

Formula for the steric and activation factors during rod grinding. The steric factor, as for ball mills, should be based on the analysis of the ratio of the fracture zone below the grinding body and the general zone covered by the projection of this body on the impact platform. To arrange the chaotic picture of falling of the grinding body on the grains, it is assumed, also for the balls, grains of dj diameter in a single layer are placed on this platform, with allowance for the possible cramping of the grains by the grinding body only for the height of the segment equal to the diameter of the grains. For a rod mill, in view of the uniform distribution of the monolayer along the rod, it suffices to be restricted to consideration to the cross-section of the rod. In this case, the scheme of the grain destruction zone remains, also for the ball (Figure 1).



d_r – rod diameter, d_j – grain diameter, h – circle segment arrow
The grains are shaded in the zone of destruction, not shaded - in the “dead” zone

Figure 1 - Ratio of fracture zones and screening of grains in the cross-section of a rod for a rod mill

However, the geometry of the destruction zone will be described not by a spherical but by a flat segment, and the width of the cramping zone will be expressed by the formula for the chord a:

$$a = 2 \cdot \sqrt{d_j d_r - d_j^2}\ , \tag{4}$$

and the steric factor is described by the ratio:

$$P_{st.r} = \frac{a}{d_r} = \frac{2 \cdot \sqrt{d_j d_r - d_j^2}}{d_r} = \tag{5}$$

$$= \sqrt{4d_j\left(d_r - d_j\right)/d_r^2},$$

where r-index refers to rod characteristics.

Comparing the formulas for the steric factor for the ball load (2) and for the rod load (5), we find that for an equal diameter of the ball and rod, the ratio $P_{st.r}$ and $P_{st.b}$ turns out to be interrelated as

$$P_{st.r} = \sqrt{P_{st.b}}\ , \tag{6}$$

and taking into account the fact that they are both smaller than one, an inequality is obtained indicating a relatively larger zone of cramping of grains by the rod

$$P_{st.r} > P_{st.b}\ , \tag{7}$$

as it should be for a body with a larger specific surface, which is a rod in comparison with a ball of equal mass and equal diameter.

The particular “polymerity” (distribution over the length) of the impact of collision of the rod and the grain leads to a corresponding formula for the activation factor, different from that found for ball mills. When a ball collides with a grain, the probability of its destruction is determined by taking into account the total mass of the ball, which is reflected in formula (3). In the case of the rod shock load, an ideally ordered picture of contact with the grain should refer to a uniformly filled monolayer of grains directly under the rod, so that one grain will have a rod volume equal to the volume of a horizontal cylinder of d_j length and with a surface area of its base $\pi d_r^2/4$, which corresponds to the partial mass of the rod (per one grain):

$$m_{r,j} = \pi d_r^2 \gamma_r d_j / 4\ , \tag{8}$$

where γ_r – density of the rod material.

However, for comparison with the impact of the entire mass of a ball on a grain, it is necessary to take into account the mass of a rod that is within the volume occupied by the balls mixed with ore and water, which fill the voids in the balls packing. With ordinary packing, this corresponds to an elementary volume in the form of a cube with a ball inscribed in it. In such a cube a part of the rod of length equal to the diameter of the ball will be inscribed, provided $d_r = d_b$.

In this case, the partial mass of the rod upon collision with the grain will be expressed as

$$m_{r,j} = \pi d_r^3 \gamma_r / 4\,, \tag{9}$$

Substituting this mass instead of the mass of a grinding body in the general formula for the activation factor (3), we obtain its version with reference to rod load:

$$P_{a,r} = \exp\left(-\frac{E_a}{RT + \dfrac{3MgD\gamma_r}{4\gamma_g}\left(\dfrac{d_r}{d_j}\right)^3} \right). \tag{10}$$

To compare the activation factors for rod and ball loads, it is also necessary for the latter of them to express the ball mass in terms of volume and density:

$$m_b = \pi d_b^3 \gamma_b / 6$$

and substitute in (3):

$$P_{a,b} = \exp\left(-\frac{E_a}{RT + \dfrac{MgD\gamma_b}{2\gamma_g}\left(\dfrac{d_b}{d_j}\right)^3} \right). \tag{11}$$

The second summand in the denominators of formulas (10) and (11) is the collision energy, respectively $E_{im,r}$ и $E_{im,b}$. Their ratio, taking into account the same density γ_r and γ_b equals to

$$\frac{E_{im,r}}{E_{im,b}} = 1{,}5\left(d_r / d_b\right)^3. \tag{12}$$

Consequently, the energy of collision with a rod at $d_r = d_b$ is 1.5 times greater than for a ball.

Equality of collision energy, and hence of destructibility for the activation factor, is achieved according to formula (12) with the correlation of the diameters of balls and rods

$$d_r = \sqrt[3]{2/3}\, d_b \cong 0{,}87 d_b\,, \tag{13}$$

which indicates a higher efficiency of destruction during rod grinding, primarily for large fractions, since their initial content is always greater than that of small ones, and the process in the sand circulation mode takes only a few minutes, during which the small fractions just begin to form.

In general, the ratio of the activation factors $P_{a,r}/P_{a,b}$ will also depend on the remaining parameters in the second summand of the denominator of the fraction in the exponents (10) and (11), which can be estimated only in model calculations.

Comparative analysis of grinding process in ball and rod mills under model conditions. It was shown before [3, 4] that the counteraction of the steric and activation factors determines the extreme course of the rate constant of grinding in ball mills

$$k_j = Z P_{st} P_a P_b P_g\,, \tag{14}$$

which together with a strict formula for the output of fractions in time (P_{0j} is the initial content of fractions) reflects the complexity of the sequential conversion of large fractions into small fractions with the identification of all the features of the process

$$P_n = P_{0n} e^{-k_n \tau} + \sum_{j=1}^{n-1} P_{0j} \prod_{j}^{n-1} k_j \sum_{j}^{n} \frac{e^{-k_j \tau}}{\prod\limits_{\substack{j,i=1 \\ i \neq j}}^{n} \left(k_i - k_j\right)}. \tag{15}$$

Moreover, the integral form of the grinding model (15) is unified for any kind of this process, and all differences are concentrated only in the constant of the process rate.

For ball mills this constant has the following form:

$$k_{j,b} = \frac{8 d_j \left(d_b - d_j\right) \omega G_b G_g}{d_b^2 \left(1 + 2\omega\sqrt{2D/g}\right)\left(G_b/\gamma_b + G_g/\gamma_g + G_w/\gamma_w\right)^2 \gamma_b \gamma_g} \exp\left(-\frac{E_a}{RT + \dfrac{MgD\gamma_b}{2\gamma_g}\left(\dfrac{d_b}{d_j}\right)^3} \right),\ \mathrm{s}^{-1}, \tag{16}$$

for rod mills:

$$k_{j,r} = \frac{4\sqrt{d_j\left(d_{\mathrm{r}} - d_j\right)}\,\omega G_{\mathrm{r}} G_{\mathrm{g}}}{d_r\left(1 + 2\omega\sqrt{2D/g}\right)\left(G_{\mathrm{r}}/\gamma_{\mathrm{r}} + G_{\mathrm{g}}/\gamma_{\mathrm{g}} + G_{\mathrm{w}}/\gamma_{\mathrm{w}}\right)^2 \gamma_{\mathrm{r}}\gamma_{\mathrm{g}}} \exp\left(-\frac{E_a}{RT + \frac{3MgD\gamma_{\mathrm{r}}}{4\gamma_{\mathrm{g}}}\left(\frac{d_r}{d_j}\right)^3}\right), \ \mathrm{s}^{-1}, \quad (17)$$

where G_{b}, G_{r}, G_{g} – respectively the total mass of balls, rods and water in the mills, ω is mill rotation frequency, s^{-1}.

First of all, it is appropriate to find out the counteracting effect of the steric and activation factors under the conditions of ball and rod grinding with equal effect of concentration and frequency factors, i.e. at identical for both variants total masses of grinding, grinding bodies and water in the mill, as well as the frequency of its rotation, then, under the same conditions, to calculate the output of the fraction in time using the model (15).

For comparative analysis, first it is necessary to set equal diameter of balls and rods, especially since in practice their dimensions vary approximately in the same range: for loaded balls it's from 100-125 to 25-30 mm, and for rods it's from 125 to 40 mm [2.11]. In this case, the maximum size of the final crushed product entering the grinding should not exceed 15-20 mm for rod mills and 10-13 mm for ball mills. This can limit the upper range of grain sizes in model calculations.

As well as before [3, 4], the following typical characteristics of the mill of the Zhezkazgan washing plant 3.6 × 4.0: w = 0.267 s^{-1}, G_b = 60000 kg, γ_b = 7874 kg/m^3, G_g = 18760 kg, γ_g = 2650 kg/m^3, D = 3.36 m. For quartz ore M_{SiO_2} = 0,0601 kg/mole, $E_a = \Delta H_m$ = 9170 J/mole (by heat of fusion). Other constants: T = 298 K; R = 8.31441 J/(mole·K); g = 9.807 m/s^2.

Calculation formulas for steric and activation factors and their products in the general model conditions of wet grinding (at G_w = 7500 kg, γ_w = 1000 kg/m^3) will be expressed for a rod mill as

$$P_{\mathrm{st},r} = 2\sqrt{\frac{d_j}{d_r} - \left(\frac{d_j}{d_r}\right)^2}, \quad (18)$$

$$P_{\mathrm{a},r} = \exp\left(-\frac{9170}{2477{,}7 + 4{,}413\left(d_r/d_j\right)^3}\right), \quad (19)$$

$$\left(P_{\mathrm{st}}P_{\mathrm{a}}\right)_r = 2\sqrt{\frac{d_j}{d_r} - \left(\frac{d_j}{d_r}\right)^2}\exp\left(-\frac{9170}{2477{,}7 + 4{,}413\left(d_r/d_j\right)^3}\right), \quad (20)$$

and for a ball mill as

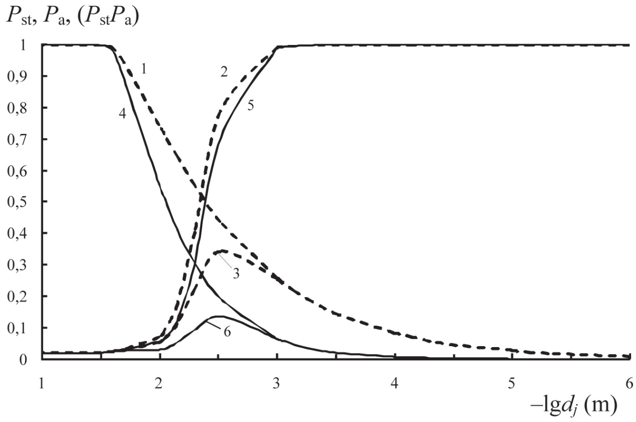
$$P_{\mathrm{st},b} = 4\left[\frac{d_j}{d_b} - \left(\frac{d_j}{d_b}\right)^2\right], \quad (21)$$

$$P_{\mathrm{a},b} = \exp\left(-\frac{9170}{2477{,}7 + 2{,}942\left(d_b/d_j\right)^3}\right), \quad (22)$$

$$\left(P_{\mathrm{st}}P_{\mathrm{a}}\right)_b = 4\left[\frac{d_j}{d_b} - \left(\frac{d_j}{d_b}\right)^2\right] \mathrm{x} \quad (23)$$

$$\mathrm{x}\ \exp\left(-\frac{9170}{2477{,}7 + 2{,}942\left(d_b/d_j\right)^3}\right).$$

The calculation results for the total size of grinding bodies $d_r = d_b$ = 0.06 m are shown in Figure 2.

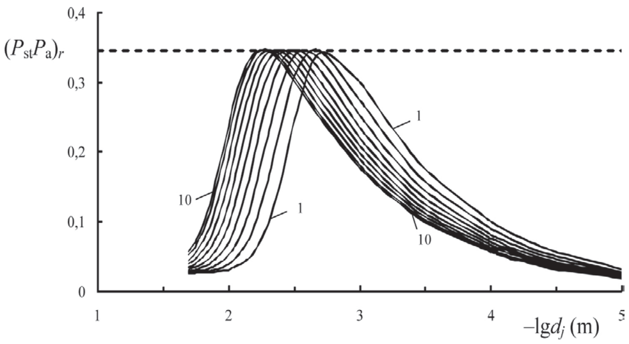


$d_r = d_b$ = 0.06 m

Figure 2 - Dependence of the steric (1, 4) and activation (2, 5) factors, as well as their products (3, 6) on the average size of the fractions for rod (1-3) and ball (4-6) grinding

From these data it follows that the steric factor for rod grinding over the whole range of grain sizes exceeds that for ball grinding, least shielding the grains from the direct hit of the grinding body. At the same time, the activation factor has a similar superiority, especially in the field of large fractions, beginning with millimeters. The combined effect of both factors leads to the formation of a maximum in the area of millimeter fractions, which is much higher for rod grinding than for ball grinding (0.342 against 0.134). In view of this, in general, the process of grinding by rods is more effective, as it corresponds to the data of practice (energy consumption is less by 25 % [11]).

It was previously established that the maximum of $P_{st}P_a$ for ball grinding displays itself at the level of 0.134 when using any ball sizes, and serves as a kind of invariant of the maximum efficiency of this process [7]. A similar invariant is found at a variation in a diameter of rods, but at a higher level, equal to 0.342 (Figure 3).



Curves 1-10 - for d_r from 0.040 to 0.125 m

Figure 3 - Dependence of the combined effect of the steric and activation factors $P_{st}P_a$ on the diameter of the rod d_r and grains d_j

In addition, as the diameter of the rod increases, as the diameter of the ball does, the maximum shifts toward larger fractions, but a much larger value of $P_{st}P_a$ falls to their share, which determines preferability of grinding such fractions by rods. Moreover, their content in a relatively short grinding period remains dominant, preparatory for subsequent grinding in ball mills.

More directly, a representation of the efficiency of rod grinding of large grains can give the results of calculation using the integral formula (15) with the rate constant (17), which, taking into account the general model conditions for wet grinding, will take the following form:

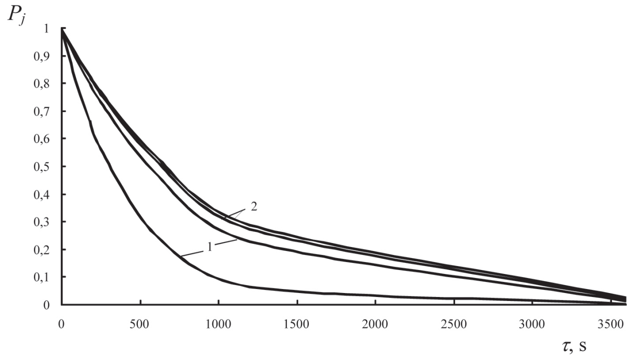
$$k_{j,r} = 8{,}107 \cdot 10^{-2} \sqrt{\frac{d_j}{d_r} - \left(\frac{d_j}{d_r}\right)^2} \exp\left(-\frac{9170}{2477{,}7 + 4{,}413\left(d_r/d_j\right)^3}\right),\ \mathrm{s}^{-1} \quad (24)$$

and for ball grinding under the same conditions when substituting them in (16):

$$k_{j,b} = 0{,}16214\left[\frac{d_j}{d_b} - \left(\frac{d_j}{d_b}\right)^2\right] exp\left(-\frac{9170}{2477{,}7 + 2{,}942\left(d_b/d_j\right)^3}\right),\ \mathrm{s}^{-1} \quad (25)$$

Detailed calculations for comparable grinding bodies of the same size showed that in the case of rod grinding, distribution of the fractions is more smooth with less significant maxima due to more efficient fracturing of all fractions. The absolute value of the maximum is approximately 1.4 times smaller than in the case of ball milling. In addition, during rod grinding, this maximum is shifted towards smaller fractions. Both features fully correspond to the practice data on a more uniform (less contrast) grinding in rod mills [2, 11, 12]. This determines the greater advisability of using ball mills in the mode of returning sands in cycles of circulation, because of the greater contrast of fractional composition in the initial stage of grinding. However, the advantage in speed of crushing the largest fractions determines preference of rod mills.

For a direct comparison of the distribution of fractions in the rod and ball grinding of the largest class with d_1 = 0.02 m, we use the equality of the diameters of both grinding bodies in two versions under the accepted model conditions: at $d_r = d_b$ = 0.125 m and $d_r = d_b$ = 0.04 мм. The results are shown in Figure 4.



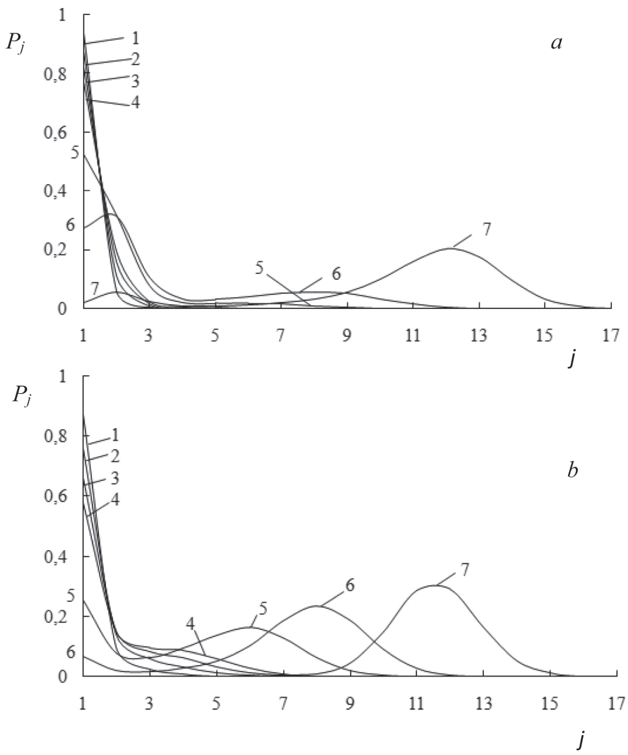
1 – for $d_r = d_b$ = 0.125 m, 2 – for $d_r = d_b$ = 0.04 m
lower curves 1 and 2 for rod grinding, upper curves for ball grinding

Figure 4 - Dependence of class content with d_1 = 0.02 m on grinding time

Here, when using rods, their obvious advantage is revealed for crushing the largest fraction. This advantage is especially accurately realized for large grinding bodies with $d_r = d_b$ = 0.125 m, at which the difference in outputs of fractions reaches 20-25 % of abs. For smaller grinding bodies with $d_r = d_b$ = 0.04 m, this difference becomes negligibly small, 1.0-1.5 %

of abs. And taking into account the better contrast of fractional composition in ball grinding, this variant of the process becomes preferable.

It should be noted that the complete picture of the change in fractional composition over time for rod mills, as well as for ball mills, is accompanied by gradual formation of a logarithmically normal distribution of fractions by condition $d_{j+1} = d_j/2$ (Figure 5).



Time of grinding, s: 1 – 60, 2 – 120, 3 – 180, 4 – 240, 5 – 600, 6 – 1200, 7 – 3600
Diameter of a rod, m: *a* = 0.04; *b* = 0.125

Figure 5 - Dependence of fractional composition (P_j, unit fr.) on the multiplicity of j and the time of grain grinding

Conclusions. The proposed probabilistic model of rod grinding reflects adequately the real features of this process and can be used for further development and application.

A comparative analysis of grinding process in ball and rod mills using a probabilistic model has confirmed the preference for rod grinding for larger fractions due to the advantage of the steric factor due to less screening of grains by a rod than by a ball. To a somewhat lesser extent, this refers to the influence of the activation factor, taking into account the additional destructive load along the length of the rod. In view of this, on the whole, the process of grinding by rods is more effective in comparison with ball grinding of large fractions, as it corresponds to the practice data, which is characterized by smaller power consumption by 25 %. At the same time, the calculated distribution of fractions becomes more uniform (less contrast), which is also a feature of rod grinding in comparison with ball grinding [13].

In rod grinding, as well as in ball grinding, a logarithmically normal distribution of fractions is formed as the process proceeds, in view of submission of the same integral model for the output of each fraction at any time.

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ТҮЙІНДЕМЕ

Жұмыстың мақсаты белсендіргіш, стерикалық, шоғырлық және жиіліктік факторлардың осы үрдіске бейімделуімен сырықты диірмендерде ұсақтау үрдістерінің ықтималдық теориясының бейнесінен тұрады. Ықтималдық үлгі негізінде шарлы және сырықты диірмендерде ұсақтау үрдісіне салыстырмалы талдауы жасалды, жәнеде талдау жасау барсында сырықты диірмендерді үлкен фракцияларды ұсақтау үшін пайдаланған қолайлы екені анықталды. Бұл стерикалық факторда басымдылық есебінен жетеді. Түйірлерді шардан гөрі сырықпен экрандаудың аз болғандығынан, сырықты ұсақтау үшін түйіршік өлшемдерінің барлық диапозонында шарлы ұсақтау үшін ол одан асып түседі. Сол мезетте белсендіргіш фактор дәл осындай басымдылықпен ерекшеленеді, турасында бірнеше төмен дәрежеде, жәнеде ол өзін ірі фракциялар үшін қатты көрсетеді. Сырықты және белсендіргіш факторлардың бірлесіп әсер етуі милиметрлік фракциялардың аймағында максимумдардың қалыптасуына әкеліп соқтырады. Бұл максимум шарлы ұсақтауға қарағанда сырықты ұсақтау үшін әлде қайда жоғары. Осыған байланысты теориялық тұрғыда сырықтармен ұсақтау үрдісі ірі фракцияларды шарлы ұсақтауға қарағанда әлде қайда тиімді болып , және электрэнергияны аз жұмсалуына сай келеді. Есептеулер сырықты ұсақтауда фракцияларды бөліп тарату бірқалыпты болатынын, жәнеде тәжірибелік мәлеметтермен келісілетінін көрсетті. Шарлы ұсақтау сияқты, сырықты ұсақтауда да үрдістің өту мүмкіндігі бойынша фракциялардың логарифметикалық бөліп таратылуы қалыптасады, және де заттектердің жүйелі бұзылуының кез келген варианттарына ұсақтаудың интегральды үлгісін бірлесе пайдаланылуымен байланысты. Алынған сырықты ұсақтаудың ықтималдық үлгісі барлық әрекеттегі факторлар есебінің арқасында тәжірибелік пайдалану үшін дайын және толық деп есептеліне алады.

Түйінді сөздер: ықтималдық үлгі, зерттеме, ұсақтау, сырықты диірмендер, шарлы диірмендер, стерикалық фактор, белсендіргіш фактор, салыстырмалы талдау.

РЕЗЮМЕ

Цель работы состоит в отображении вероятностной теорией измельчения процессов в стержневых мельницах с адаптацией к этому процессу частотного, концентрационного, стерического и активационного факторов. Проведен сравнительный анализ процесса измельчения в шаровых и стержневых мельницах на основе вероятностной модели, в ходе чего выяснено, что стержневые мельницы предпочтительнее использовать для измельчения более крупных фракций. Это достигается за счет преимущества в стерическом факторе. Он для стержневого измельчения во всем диапазоне размеров зерен превосходит таковой для шарового измельчения, ввиду меньшего экранирования зерен стержнем, чем шаром. В то же время и активационный фактор отличается подобным же превосходством, правда в несколько меньшей степени, который особенно сильно проявляет себя для крупных фракций. Совместное воздействие стерического и активационного факторов приводит к формированию максимумов в области миллиметровых фракций. Этот максимум существенно выше для стержневого измельчения нежели для шарового. Ввиду этого процесс измельчения стержнями является теоретически гораздо более эффективным, чем шаровое измельчение крупных фракций, что соответствует практически меньшему расходу электроэнергии. Расчеты показали, что при стержневом измельчении распределение фракций получается более равномерным, и это также согласуется с практическими данными. При стержневом измельчении, также как и при шаровом, формируется логарифмически нормальное распределение фракций по мере протекания процесса, что связано с единством применимости интегральной модели измельчения к любым вариантам последовательной деструкции вещества. Полученная вероятностная модель стержневого измельчения благодаря учету всех действующих факторов может считаться наиболее полной и готовой для практического использования.

Ключевые слова: вероятностная модель, разработка, измельчение, стержневые мельницы, шаровые мельницы, стерический фактор, активационный фактор, сравнительный анализ

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